Name:_____ Math 260
Date: 3/11/2025 Exam 1

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

1. (20 points) Solve the following systems of equations by row reducing an augmented matrix to reduced row-echelon form (Calculator OK)

a) $$\begin{aligned} x_1 + 3x_2 + 2x_3 + 16x_4 &= 0 \\ 3x_1 + 9x_2 - x_3 + 20x_4 &= 0 \\ -2x_1 - 6x_2 + 6x_3 + 8x_4 &= 0 \\ 2x_1 + 6x_2 - x_3 + 12x_4 &= 0 \end{aligned}$$

b) $$\begin{aligned} 4x_1 + 2x_2 + 5x_3 &= -3 \\ 2x_1 - 5x_2 + 3x_3 &= 7 \\ 2x_1 + 19x_2 + x_3 &= 2 \end{aligned}$$

2. (20 points) Balance the chemical reaction below by solving a system of equations. (Calculator OK)

$$C_8H_{18} + O_2 \rightarrow CO_2 + H_2O$$

3. (20 points) If $A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 8 \\ -2 & 4 \end{bmatrix}$, find… (By hand, no calculator)

a) $3A - 2B$

b) $B^{-1}A^{T}$

4. (20 points) Use the matrix inversion algorithm to find A^{-1} if $A = \begin{bmatrix} 3 & 5 & 0 \\ 3 & 7 & 1 \\ 1 & 2 & 1 \end{bmatrix}$. (By hand, no calculator)

5. (25 points) Write $A = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$ as a product of elementary matrices

6. (20 points) Prove each of the following…

a) If A and B are $m \times n$ matrices, then $(A + B)^T = A^T + B^T$

b) If A and B are an $n \times n$ invertible matrices, then so is AB and $(AB)^{-1} = B^{-1}A^{-1}$

7. (20 points) Prove or disprove each of the following…

a) If E is an elementary matrix, then $\det E = 1$

b) $\forall\, x \in \mathbb{R}$, if $\sin x = 1$ then $\cos x = 0$.

8. (15 points) Use mathematical induction to prove: $\forall\, n \in \mathbb{N}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$